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But

$$S_{3/2}(4N, \chi\varepsilon_{4N}) \xrightarrow{\simeq} S_{3/2}(4N,\chi)^*$$

$$f \longmapsto \langle \cdot, \iota W_{4N} f \rangle$$

Notation: $\varepsilon_{4N} = \left(\frac{4N}{\cdot}\right)$,

$$(W_{4N} f)(\tau) = f\left(\frac{-1}{4N\tau}\right)(-i\sqrt{4N}\tau)^{-3/2}$$

$$(\iota f)(\tau) = \overline{f(-\bar{\tau})}$$

Thus we obtain a map

$$L_{t,\chi} : C(2N,\chi^2) \longrightarrow S_{3/2}(4N, \chi\varepsilon_{4N})$$

$$\langle g, \iota W_{4N} L_{t,\chi}(c) \rangle = \int_{c^+} S_{t,\chi}(g) \qquad \circledast$$

Theorem 1) The map $L_{t,\chi}$ defined by $\circledast$ commutes with all $T(p)$ $(\gcd(p,4N)=1)$.

2) $\operatorname{Im} L_{t,\chi} = W_{4N} \circ \iota\, (\operatorname{Ker} S_{t,\chi})^{\perp}$

3) $\sum_t L_{t,\chi} = S_{3/2}(4N, \chi\varepsilon_{4N})$ (since $\sum_t L_{t,\chi} = W_{4N}\circ\iota\,(\bigcap \operatorname{Ker} S_{t,\chi})^{\perp}$, $\bigcap = 0$)

Recall

$$S_{t,\chi}(g) = \langle g, \Theta_{t,\chi}(\cdot, z) \rangle$$

Hence

$$\langle g, \iota W_{4N} L_{t,\chi}(c) \rangle = \int_{c^+} \langle g, \Theta_{t,\chi}(\cdot, z) \rangle$$

$$= \langle g, \int_{c^+} \Theta_{t,\chi}(\cdot, z)\, d\bar{z} \rangle$$

(Interchanging $\int_{c^+}$ and volume integral can be justified)