同濟大學
TONGJI UNIVERSITY
SHANGHAI,PEOPLE'S REPUBLIC OF CHINA

⌈ Follows from the short exact sequence of $\Gamma_0(m)$-modules

$$0 \to \mathbb{C} \to \mathrm{Hom}(\mathbb{Z}[\mathbb{P}^1(\mathbb{Q})],\mathbb{C}) \xrightarrow{\mathrm{res}} \mathrm{Hom}(\mathbb{Z}[\mathbb{P}^1(\mathbb{Q})]^0, \mathbb{C}(\chi)) \to 0$$

$$z \mapsto \text{“}c \mapsto \deg(c)\, z\text{”}$$

which induces a long exact sequence

$$\cdots \square \to \mathrm{Hom}_{\Gamma_0(m)}(\mathbb{Z}[\mathbb{P}^1(\mathbb{Q})]^0, \mathbb{C}(\chi)) \xrightarrow{\delta} H^1(\Gamma_0(m), \mathbb{C}(\chi)) \to \square \cdots ⌋$$

Let

$C(m,\chi)$ = dual of right hand side (without "eval")

Then

$$\boxed{C(m,\chi) = \ker\left([\mathbb{Z}[\mathbb{P}^1(\mathbb{Q})]^0 \otimes \mathbb{C}(\bar\chi)]_{\Gamma_0(m)} \to [\mathbb{Z}[\mathbb{P}^1(\mathbb{Q})] \otimes \mathbb{C}(\bar\chi)]_{\Gamma_0(m)}\right)}$$

(induced by

$$\mathrm{Hom}_{\Gamma_0(m)}(\mathbb{Z}[\mathbb{P}^1(\mathbb{Q})]^0, \mathbb{C}(\chi)) \to \mathrm{Hom}_{\Gamma_0(m)}(\mathbb{Z}[\mathbb{P}^1(\mathbb{Q})]^0 \otimes \mathbb{C}(\bar\chi), \mathbb{C}(1)))$$

$$\lambda \mapsto \varphi_\lambda \qquad \varphi_\lambda(c \otimes z) = \lambda(c) \cdot z \quad)$$

$$T(\ell)[D] = \sum_i [r_i D]$$

§3 Parametrization of $S_{3/2}(4N,\chi)$

Consider

$$S_{3/2}(4N,\chi) \xrightarrow[S_{t,\chi}]{} S_2(2N,\chi^2) \xrightarrow[f \mapsto \lambda_f]{} C(2N,\chi^2)^*$$

and dualize to obtain a Hecke-equivariant

$$L_{t,\chi} : C(2N,\chi^2) \to S_{3/2}(4N,\chi)^*$$

$$L_{t,\chi}(c) = \text{“}\int_c\text{”} \circ S_{t,\chi}, \text{ i.e.}$$

$$L_{t,\chi}(c)(f) = \int_{c^+} S_{t,\chi}(f) .$$