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One has

$$S_{t,\chi}(f)(z) = \langle f, \Theta_{t,\chi}(\cdot, z)\rangle \qquad (z \in \mathfrak{H})$$

with a certain "$\Theta$-kernel" (later)

(Note: $\Theta_{a,\chi}(\tau, \cdot)$ transf. like elem. of $\overline{S_2(4N, \chi^2)}$ ).

Thm $S_{3/2}(4N,\chi)$ = orthogonal complement of subsp. of "trivial" wt forms
(f trivial: $\exists t_1 \ldots t_r$ s.t. $a_f(n) = 0$ unless $t_j n = \square$ for some $j$)

Period mapping (Manin symbols, Eichler–Shimura isomorphism ...)

The application

$$f \mapsto \lambda_f \,, \quad \lambda_f(c) := \sum_s c_s \left(\int_s^\infty + \int_{-s}^\infty\right) f(z)\, dz =: \int_{c^+} f$$

(where $c = \sum c_s (s) \in \mathbb{Z}[\mathbb{P}^1(\mathbb{Q})]^0$ )

defines an isomorphism

$$S_2(m, \chi) \xrightarrow{\approx} \frac{\mathrm{Hom}_{\Gamma_0(m)}\left(\mathbb{Z}[\mathbb{P}^1(\mathbb{Q})]^0, \mathbb{C}(\chi)\right)^{\text{even}}}{\mathrm{Hom}_{\Gamma_0(m)}\left(\mathbb{Z}[\mathbb{P}^1(\mathbb{Q})], \mathbb{C}(\chi)\right)^{\text{even}}}$$

which commutes with all Hecke operators.

Here $\mathbb{C}(\chi) = \mathbb{C}$ with $\Gamma_0(m)$-action $(A, z) = \chi(A) z$,

even: $\lambda(\sum c_s (s)) = \lambda(\sum c_s (-s))$.

Hecke action:

$$(T(\ell)\lambda)(c) = \sum_{\substack{M \in \Gamma_0(m) \backslash \begin{pmatrix} \mathbb{Z} & \mathbb{Z} \\ 4\mathbb{Z} & \mathbb{Z} \end{pmatrix}_{\det = \ell} \\ \gcd(a, m) = 1}} \chi(a)\, \lambda\left(\sum c_s (Ms)\right).$$