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On the computation of modular forms of half integral wt.

§1 Introduction

Possibilities for computing $S_{3/2}(4N, \chi)$

(assume throughout χ even, otherwise $S_{3/2}(4N,\chi) = 0$ trivially)
and χ mod $4N$, i.e. $\chi(a) = 0$ if $\gcd(a, 4N) > 1$)

(1) the Θ-series associated to ternary forms

Disadvantage:
- Θ-series do not generate all of $S_{3/2}(4N,\chi)$
- the behaviour under Hecke operators is "clumsy"

(2) Basmaji's method: (Basmaji's thesis, p. 55)

Suppose $N \equiv 0 \bmod 4$

the image of

$$S_{3/2}(4N,\chi) \longrightarrow S_2(4N,\chi) \times S_2(4N,\chi)$$
$$f \longmapsto \left(f \sum_{n\geq 1} q^{n^2/2}, f \sum_n q^{n^2}\right)$$

where $\sum_{n\geq 1} q^{n^2/2} = \Theta_2$ and $\sum_n q^{n^2} = \Theta_3$

is $\{(f_2, f_3) \in S_2(4N,\chi) \times S_2(4N,\chi) \mid f_2\Theta_3 = f_3\Theta_2\}$

Thus

basis of $S_2(4N,\chi) \rightsquigarrow$ basis of $S_{3/2}(4N,\chi)$

Disadvantage:
- only for $4 \mid N$
- needs basis of $S_2(4N,\chi)$
  (even if only certain Hecke ev's are wanted)
- not really closed formulas for Fourier coefficients

(3) the Jacobi form ...