

§ 4. Genus 11F of abelian

(11)

Example: $\Gamma^2 = \Gamma_0(11)$

$$S_2(11) = \mathbb{C} \eta(z)^2 \eta(11z)^2$$

$$\Gamma_0(11) \backslash SL(2, \mathbb{Z}) \cong \mathbb{P}^1(\mathbb{F}_{11}) = \mathbb{F}_{11} \cup \infty$$

$$\begin{pmatrix} * & * \\ c & d \end{pmatrix} \leftrightarrow [c:d] \pmod{11}$$

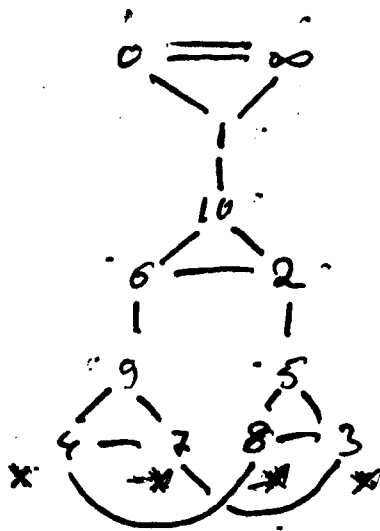
$$[a:1] \leftrightarrow a$$

$$[1:0] \leftrightarrow \infty$$

action of $SL(2, \mathbb{Z})$:
 evolution:

$$\alpha S = \frac{-1}{\alpha} \quad \alpha S^{\vee} = \frac{1}{1-\alpha}$$

$$\alpha \mapsto -\alpha$$



red 0's : $\langle S^{\vee} \rangle$ -orbits

blue 2-angles : $\langle S \rangle$ -orbits

$M^- = \mathbb{C} \cdot \lambda$, where

$$\lambda(\alpha) = \begin{cases} 1 & \text{if } \alpha = 3, 9 \\ -1 & \text{if } \alpha = -3, -9 \\ 0 & \text{otherwise} \end{cases}$$

$$\eta(z)^2 \eta(11z)^2 = \sum_{(a,b) \in \mathbb{Z}^2} \lambda\left(\frac{a}{b}\right) \eta\left(\frac{a+11bz}{b}\right)^2$$

By Mob. thm. this funk. may explicit using (1).

Hence $M^- = \mathbb{C} \cdot \lambda$, where

$$\lambda\left(\frac{s}{1} - \frac{\infty}{1}\right) = \#\{k \mid q_k \equiv \frac{3}{4} q_{k-1}\} - \#\{k \mid q_k \equiv -\frac{3}{4} q_{k-1}\}$$

($\infty = \frac{1}{0} = \frac{p_0}{q_0}, \frac{p_1}{q_1}, \dots, \frac{p_n}{q_n} = s$: convergents of
 a.f. expansion of s)

$$\eta(z)^2 \eta(11z)^2 = \sum_{l \geq 1} \lambda\left(\frac{1}{3} - \frac{0}{1}\right) q^l$$

$$= \sum_{l \geq 1} \left(\sum_{d|l} \sum_{b=0}^{d-1} \left[\lambda\left(\frac{l}{3d^2} + \frac{b}{d}\right) - \lambda\left(\frac{l}{d} - \infty\right) \right] \right) q^l$$

→ goto (18)