同濟大學 教师备课用纸

Exercise Show for $k=2$ and $\gcd(l,N)=1$, $v \in Cr_0(N)$:

(1) $$\lambda_{T(l)f}(v) = \sum_{\substack{ad=l\\ a,d>0\\ b \bmod d}} \sum_{\mu=1}^{r(b/d)} \lambda_f\left(v\begin{pmatrix} a & b\\ 0 & d\end{pmatrix}^{-1} C_{\frac{b}{d},\mu}\right)$$

where $\frac{b}{d} = [a_0, \ldots, a_r]$ $\quad r(b/d) = r$

$$C_{\frac{b}{d},\mu} = \begin{pmatrix} a_0 & 1\\ 1 & 0\end{pmatrix} \cdots \begin{pmatrix} a_r & 1\\ 1 & 0\end{pmatrix}\begin{pmatrix} 1 & 0\\ 0 & (-1)^{\mu-1}\end{pmatrix}.$$

~~Γ_{l^2} $\sum_{M \in \Gamma \backslash M(l)} M = l^2 \sum_M \int_{MA_0}^{MA_\infty} f = \sum_N \int_{AN_0}^{AN_\infty} f$~~

~~(since $MA = C_n A N_n$ for suitable $C_n \in \Gamma$, $N_n \in M(l)$ etc....)~~

~~$= l^2 \sum_{N \text{ diagonal}} \int_{AN_0}^{A_\infty} f = l^2 \sum_{\substack{ad=l\\ b \bmod d}} \int_{b/d}^{\infty} f$~~

~~$\infty - \frac{b}{d} =$~~

Define $T(l)\lambda$ by the right hand side "with f dropped".

~~Then~~ $\lambda \in X_2(\Gamma_0(N))$

Then Analyzing the "Eisenstein part" $\in X_0(\Gamma)^{\pm}$ one can prove

Thm. $X_0(\Gamma_0(N))^+$ resp. $X_0(\Gamma_0(N))^-$ are

Hecke equivariantly isomorphic to subspaces of $M^{\pm}_2(\Gamma_0(N))$,

respectively. M^+ and M^- contain $S_2(\Gamma_0(N))$ and

$M^+ + M^- = M_2(\Gamma_0(N))$.

go to (11)