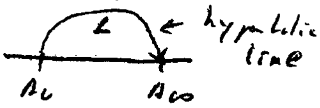
~~Then~~ unnecessary explicit maps

For $f \in S_2(\Gamma)$, $A \in SL(2,\mathbb{Z})$ set

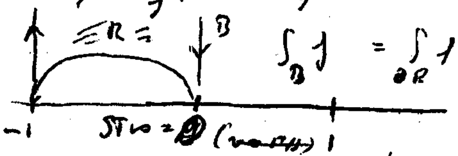
$$\lambda_f : G_\Gamma \longrightarrow \mathbb{C}, \quad \lambda_f(\Gamma A) = \int_{A0}^{A\infty} f(z)\,dz = \int_L f(z)\,dz$$



Modular symbol: $\{p,q\} \in S_k(N)^* \to$; $\{p,q\}(f) := \int_p^q f(z)\,dz$

[Note: $\lambda(CA) = \int_{CA0}^{CA\infty} f(z)\,dz = \int_{A0}^{A\infty} f(z)\,dz = \lambda(A)$ for $C \in \Gamma$].

Note
$$\lambda_f(v) + \lambda_f(vS) = 0, \quad \lambda_f(v) + \lambda_f(vR) + \lambda_f(vR^2) = 0$$



$\int_B f = \int_{\partial R} f - \int_R df = 0$

(where $\lambda_f^\pm$ the projection on $X_2(\Gamma)^\pm$ along $X_2(\Gamma)^\mp$)

Thm $f \to \lambda_f^+$ resp. $f \mapsto \lambda_f^-$ define injections $S_2(\Gamma) \longrightarrow X_2^+(\Gamma),\ X_2(\Gamma)^-$

Similarly for $k \geq 2$, $f \in S_k(\Gamma)$, $A \in SL(2,\mathbb{Z})$
$$\lambda_f(A) := \int_{A0}^{A\infty} f(z)(X - zY)^{k-2}\,dz$$

Exercise: $\lambda_f(GA) = G.\lambda_f(A)$ for $G \in \Gamma$.
Hint: $(Aw - Az)(cw+d)(cz+d) = w - z$.

goto (9) ⟶

Thm For each Γ there are functorial injections
$$\lambda^\pm : S_k(\Gamma) \hookrightarrow X_k(\Gamma)^\pm.$$

For $\Gamma = \Gamma_0(N)$ these maps become $\mathbb{T}(N)$-module injections if one defines Hecke operators $T(l)$ on $X_k(\Gamma)$ via (see (1) on other page).