$\Gamma = \Gamma_0(p)$, p prime

f.pts of S on $C_p = \mathbb{P}^1(\mathbb{F}_p)$: $\quad \frac{-1}{a} = a$, i.e. $a^2 = -1$

$$\# = 1 + \left(\frac{-4}{p}\right)$$

f.pts of R on ~~C_p~~ $= \mathbb{P}^1(\mathbb{F}_p)$: $\quad \frac{1}{1-a} = a$, i.e. $a^2 - a + 1 = 0$

$$\# = 1 + \left(\frac{-3}{p}\right)$$

$$d_{\Gamma_0(p)} = \frac{p+1}{6} - \frac{1}{2}\left(1 + \left(\frac{-4}{p}\right)\right) - \frac{2}{3}\left(1 + \left(\frac{-3}{p}\right)\right) + 1$$

$$= 1 + 2 \text{ genus of } X_0(p)$$

__Ex__ For arbitrary Γ (with $-1 \in \Gamma$):

$$\boxed{d_\Gamma = \dim M_2^{Eis}(\Gamma) + 2 \dim S_2(\Gamma).}$$

~~[Complete the above calculation and compare the result to Shimura's book formula for $\dim M_2(\Gamma)$.]~~

Ex Prove

You Need

__Labels of higher weight__

$SL(2,\mathbb{Z})$ acts on $\mathbb{C}[X,Y]_n$ via $(Af)(X,Y) = f((X,Y)A^{-1})$

admissible labels: pairs (A, f) such that

- $A \in v$ (i.e. A represents the coset $v \in \Gamma\backslash SL(2,\mathbb{Z})$)
- $f \in \mathbb{C}[X,Y]_n$
- v_1, v_2 joined blue, say $v_1 = (A,f)$, $v_2 = (GAS, g)$, $G \in \Gamma$, then $f + Gg = 0$
- $v_1 v_2 v_3$ joined red, say $= (A,f), (GAR, g), (HAR^2, h)$ $(G, H \in \Gamma)$, then $f + G\cdot g + H\cdot h = 0$.

~~Ex? Redo the above for $\Gamma_0(p)$ for $\Gamma_0(N)$ (N general)~~. Result:

$$\dim M_k(N) = \frac{k-1}{12}\#\mathbb{P}^1(\mathbb{Z}/N\mathbb{Z}) - \frac{1}{4}\left(\frac{-4}{k-1}\right)\sum^*\left(\frac{-4}{d}\right) - \frac{1}{3}\left(\frac{k-1}{3}\right)\sum^*\left(\frac{d}{3}\right) + \sum^*\varphi(d)$$

$\sum^*$: $d \mid N$, N/d □-free, $Q(d)$ = largest t s.t. $t^2 \mid d$ ($\varphi\,\gcd(d)$)

$\dim E_k(N) = \#\Gamma_0(N)\backslash\mathbb{P}^1(\mathbb{Q}) - 1 = \sum^* Q(d) - 1$