同济大学 教师备课用纸

<u>Dimension calculation</u> Extend $\lambda \in X_2(\Gamma)_{\mathbb{C}}$ to $\mathbb{C}[G_p]$, a functional, using the [illegible], identify $X_2(\Gamma)_{\mathbb{C}}$ with $\mathbb{C}[G_p]^*$.

$SL(2,\mathbb{Z})$ acts on $\mathbb{C}[G_p]$, $X := \mathbb{C}[G_p]^*$

$$X_2(\Gamma)_{\mathbb{C}} = \ker(1+S) \cap \ker(1+R+R^2) \subseteq X$$

$$X_2(\Gamma)_{\mathbb{C}}^* = \ker(1+S)^* + \ker(1+R+R^2)^* \subseteq X^* = \mathbb{C}[G_p]$$

$$= \ker(1-S) + \ker(1-R) \subseteq X^*$$

[Note: $f \in \mathrm{End}(V)$, $\operatorname{ord} f = n$

$\ker(g)^* = \{ y \in V^* \mid (x,y) = 0 \ \forall x \in \ker g \}$ $\quad ((x,y) := y(x))$

$(1+f+\dots+f^{n-1} =: g)$ $\quad = \operatorname{Im} g^* = \ker(1-f)$.]

$$d_p = \dim X^* - (\dim \ker(1-S) + \dim \ker(1-R)) + \dim \underbrace{\ker(1-S) \cap \ker(1-R)}_{\subseteq X^*}$$

$$= \dim X^* - \dim X^{*S} - \dim X^{*R} + \dim X^{*SL(2,\mathbb{Z})}$$

$$= \# G_p - \dim \mathbb{C}[G_p]^S - \dim \mathbb{C}[G_p]^R + 1$$

[Note: $\mathbb{C}[G_p]^{SL(2,\mathbb{Z})} = \mathbb{C} \sum_{v \in G_p} (v)$ since $SL(2,\mathbb{Z})$ acts transitively on G_p]

$$\mathbb{C}[G_p]^S = \mathrm{span}_{\mathbb{C}} \left\langle \sum_{v \in \sigma} (v) \;\middle|\; \sigma \in G_p/\langle S \rangle \right\rangle$$

$\dim \mathbb{C}[G_p]^S = \frac{1}{2} \# G_p + \frac{1}{2} \#$ fixed pts of S on G_p

Similarly

$\dim \mathbb{C}[G_p]^R = \frac{1}{3} G_p + \frac{2}{3} \#$ fixed pts of R on G_p

$d_p = \frac{1}{6} \# G_p - \frac{1}{2}$ f.pts of S on G_p $- \frac{2}{3}$ fixed pts of R on G_p $+ 1$