



Labels

$X_2(G)$ Labels on G such that

sum of labels at vert. joined by same color = 0

Labeling: map $\lambda: G \rightarrow \mathbb{Z}$

$$X_2(G) := \left\{ \lambda: G \rightarrow \mathbb{Z} : \begin{array}{l} \lambda(v) + \lambda(vS) = 0 \\ \lambda(v) + \lambda(vR) + \lambda(vR^2) = 0 \\ \text{for all } v \in G \end{array} \right\}$$

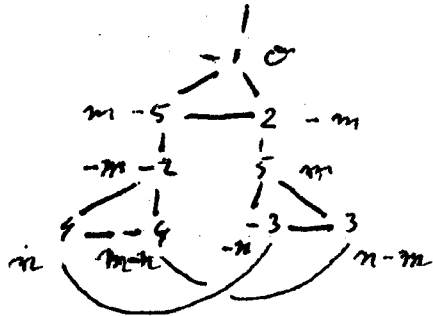
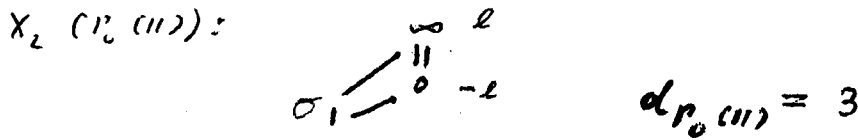
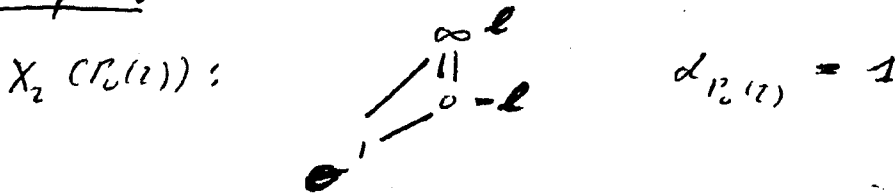
Note: $X_2(G)$ free \mathbb{Z} -module ($\cong \mathbb{Z}^{\#G}$)

• Set $X_2(G)_R = \dots \lambda: G \rightarrow R : \dots$

then $X_2(G)_R = R \otimes X_2(G)$.

$$d_G := \text{rank } X_2(G) = \dim_{\mathbb{Q}} X_2(G)_{\mathbb{Q}} = \dim_{\mathbb{C}} X_2(G)_{\mathbb{C}}$$

Examples:



Def. $X_2(N) := X_2(P_2(N))$

Thm $\dim X_2(N) = d_{E_2(N)} + 2 \dim S_2(N)$.