



Computation of modular forms

Graphs

via periods

§ 2. Labelled Schreier

$\Gamma \subseteq SL(2, \mathbb{Z})$, assume $\boxed{-1 \in \Gamma}$

coset graphs of
modular symbols

Graph G_Γ :

Vertices: $\Gamma \backslash SL(2, \mathbb{Z})$

Blue edges: $\{\Gamma A, \Gamma B\} \iff \Gamma A S = \Gamma B$

Red edges: $\{\Gamma A, \Gamma B\} \iff \Gamma A R = \Gamma B \sim \Gamma A R^{-1} B \notin$

$[S = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, T = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}, R = ST = \begin{pmatrix} 1 & -1 \\ 0 & 1 \end{pmatrix}, R^2 = \begin{pmatrix} -1 & -1 \\ 0 & 1 \end{pmatrix}$
 $S^2 = -id, R^3 = -id]$

Examples

$\Gamma_0(N) \backslash SL(2, \mathbb{Z}) \xrightarrow{\cong} \mathbb{P}^1(\mathbb{Z}/N\mathbb{Z}) = \frac{\{(x, y) \in \mathbb{Z}/N\mathbb{Z} \mid \gcd(x, y) = 1\}}{(\mathbb{Z}/N\mathbb{Z})^2}$
 $\Gamma_0(N) A \xrightarrow{\uparrow} [0:1]A = [c:d]$
 $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$

$\Gamma_0(N) A \leftrightarrow [c:d], \Gamma_0(N) X \leftrightarrow [c:d]X$

$[x:y] S = [y:-x]$

$[x:y] R = [y:-x+y], [x:y] R^2 = [-x+y, -x]$

$N = p = \text{prime}$

$\mathbb{F}_p \hookrightarrow \mathbb{P}^1(\mathbb{Z}/p\mathbb{Z}), \infty := [1:0], \mathbb{P}^1(\mathbb{F}_p) = \mathbb{F}_p \cup \infty$
 $a \mapsto [a:1]$

$a S = -1/a$ (~~$1/a$~~) $0 S = \infty, \infty S = 0$ $\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$
 $a R = 1/(1-a), a R^2 = 1 - 1/a$ $\infty \xrightarrow{R} 0 \xrightarrow{R} 1$

So graph starts always like

except for $N=1$:

