Furthermore

Prop. 3.2 on page 92

$$\Theta_{\underline{L}} |_{\frac{r}{2}} \begin{pmatrix} -1 & 0 \\ n & -1 \end{pmatrix} \underset{\text{"gives"}}{\approx} \frac{(-1)^{r/2}}{|L^\#/L|} \bar{\mathcal{Q}}(\underline{M}^\#, n) \quad (\text{Eichler pairs})$$

Thm Assume $\Theta_{\underline{L}} = \Theta_{\underline{M}}$ (and $\underline{L}$ and Δ of odd level ~~$|L^\#/L|$ odd~~)

Then $\mathcal{D}_{\underline{L}} \approx \mathcal{D}_{\underline{M}}$ (as fin. q. mod.),

$\omega_{\underline{L}} \approx \omega_{\underline{M}}$ (as $SL(2,\mathbb{Z})$-repr.).

Then $\Theta_{\underline{L}}$ stronger than $\mathcal{D}_{\underline{L}}, \omega_{\underline{L}}$ (e.g. $\underline{L}$ unimodular, then $\Theta_{\underline{L}} \neq \Theta_{\underline{M}}$ but $\mathcal{D}_{\underline{L}} = \mathcal{D}_{\underline{M}} = 0$)

Thm (Wall, reproved by Kneser)

$\underline{L} \longrightarrow \mathcal{D}_{\underline{L}}$ induces isomorphism of semigroups

$$\left\{ \begin{matrix} \text{stable iso classes} \\ \text{of even lattices} \\ \text{(not nec. p.d.)} \end{matrix} \right\} \longrightarrow \left\{ \begin{matrix} \text{iso classes of} \\ \text{fin. quad. modules} \end{matrix} \right\}$$

($\underline{L} \underset{st.}{\approx} \underline{M}$ iff $\underline{L} \oplus U_1 \approx \underline{M} \oplus U_2$ U_1, U_2 unimod. even)

Thm $\Theta_{\underline{L}} = \Theta_{\underline{M}}$ (and lattices of odd level), then

$\underline{L} \underset{st.}{\approx} \underline{M}$.