They are related by the following:

$\underline{A} = (A, Q)$ finite quadratic module,

(i.e. A finite abel. grp., $Q: A \to \mathbb{Q}/\mathbb{Z}$ s.t. $Q(na) = n^2 Q(a)$ $\forall a \in A, n \in \mathbb{Z}$,
$B(a,b) = Q(a+b) - Q(a) - Q(b)$ $\mathbb{Z}$-bilinear,
B non degenerate)

set

$$\chi(\underline{A}, n) := \sum_{x \in A} e(n Q(x)) \qquad (n \in \mathbb{Z})$$

Thm (?) $\underline{A} \cong \underline{B}$ (as quadr. modules)
iff
$\chi(\underline{A}, \cdot) = \chi(\underline{B}, \cdot)$ and $A(2) \cong B(2)$ (as abelian grps) ~~the elementary divisors of the 2-parts of A and B~~

To $\underline{A}$ finite quadr. mod. one can associate a Weil representation $\omega_{\underline{A}}$ as above for $\omega_{\underline{L}} = \omega_{D_{\underline{L}}}$.

Easy: $\underline{A} \cong \underline{B}$ then $\omega_{\underline{A}} \cong \omega_{\underline{B}}$.

As consequence of the above thm:

Thm (?) Assume $\omega_{\underline{A}} \cong \omega_{\underline{B}}$ (as $SL(2,\mathbb{Z})$-repres.) ~~and dim ω_A odd~~ Then $\underline{A} \cong \underline{B}$ as quadratic mod.

[$\chi(\underline{A}, n) = \mathrm{tr}\left(\begin{pmatrix}1 & n\\ 0 & 1\end{pmatrix}, \mathbb{C}[A]\right)$; Furthermore $\mathrm{tr}(A, \mathbb{C}[A]) = \pm \# A[a-1]$ for $A = \begin{pmatrix}a & b\\ c & d\end{pmatrix}$, level of $\underline{A}$, which yields the structure of $A(2)$.]

Thus $Q_{\underline{L}}$ or $\omega_{\underline{L}}$ equivalent as invariants of lattices.

Remark:

Thm (Nobs, Wohlfahrt) Every irred. repr. of $SL(2,\mathbb{Z})$ which factors through a repr. of $SL(2,\mathbb{Z}/N\mathbb{Z})$ is contained in some $\omega_{\underline{A}}$.