同濟大學 教师备课用纸

2) For rank 32, ~~40, 48~~ there are several non-isom. even extremal unimod. lattices ~~of rank 64~~, but their theta series are of course the same

(Recall: $f \in M_{4k}(SL(2,\mathbb{Z}))$ extremal if

$$f = 1 + O(q^{[k/3]+1}) \qquad ([k/3]+1 = \dim M_{4k}(\;))$$

f exists and unique $\qquad$ (basis of M_{4k}: $E_4^{k}, E_4^{k-3}\Delta, \dots, E_4^{k-3\ell}\Delta^{\ell}$, $\ell = [\frac{k}{3}]$)

and $\underline{L}$ are extremal unimod. if $\Theta_{\underline{L}} = f$)

In all instances this is the most that can happen.
To discuss $\Theta_{\underline{L}}$ in general we introduce two other invariants:

<u>discriminant module:</u> $\mathcal{D}_{\underline{L}} = (L^{\#}/L, \underline{Q})$

$$L^{\#} = \{ y \in \mathbb{Q}\otimes L \mid B(y,L) \subseteq \mathbb{Z} \} \supseteq L$$

$$\underline{Q} : L^{\#}/L \to \mathbb{Q}/\mathbb{Z}, \quad x+L \mapsto Q(x) \cancel{+\mathbb{Z}}$$

$\mathcal{D}_{\underline{L}}$ is a finite quadratic module. ($\underline{M} = (M,Q)$, M ab. gr., $Q: M \to \mathbb{Q}/\mathbb{Z}$ s.t. $Q(x+y)-Q(x)-Q(y)$ bil. nondeg., $Q(ax)=a^2Q(x)$)

<u>Thm</u> Suppose even. Let ℓ = level of $\mathcal{D}_{\underline{L}}$ (i.e. smallest nat. $\ell>0$ s.t. $\ell Q \equiv 0$), assume r even, let $\Delta = (-1)^{r/2}|L^{\#}/L|$. Then

$$\Theta_{\underline{L}} \in M_{r/2}(\Gamma_0(\ell), (\tfrac{\Delta}{\cdot})).$$

1) finite dim.
2) $\ell=1$, then $M_{r/2}$ as above
3) $\ell>1$ similar descriptions

~~Analogously~~ $\mathfrak{H}$, $SL(2,\mathbb{Z})$ action, actually, $Hol(\mathfrak{H})$

$M_k(\Gamma,\chi) = \{ f \in Hol(\mathfrak{H}) \mid f|_k A = \chi(A) f \;\forall A \in \Gamma \}$ + ~~s.t. ... cond.~~ cusp. cond.