Problems for a take home exam.

I. (No partial credit)

1). Let X,Y be metric spaces with metrics d_x (and d_y resp.), let $f : X \to Y$ be a map, and $x_o \in X$.

Prove: f is continuous at x_o $\Longleftrightarrow$ $\forall\, \varepsilon > 0\ \exists\, \delta > 0$ such that for all x with $d_x(x_o,x) < \delta$ we have $d_y(f(x_o),f(x)) < \varepsilon$.

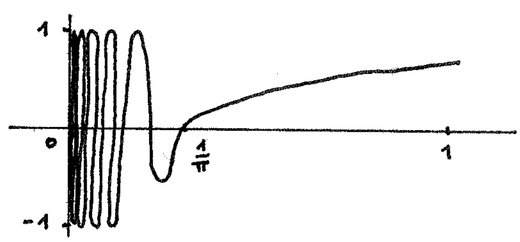
2). Solve problem 2). on page 2 of the lecture notes.

3). Show that each metric space is regular.

II. (Partial credit)

4). Prove: Each pathwise connected topological space is connected.

Remark: The converse is not true. The subspace



$$\{(x,y) \in \mathbb{R}^2 \mid (x \in (0,1) \text{ and } y = \sin\tfrac{1}{x} \text{ or } (x = 0 \text{ and } y \in [-1,1])\}$$

of the plane is connected, but not pathwise connected. However we have:

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5). Prove: A manifold is pathwise connected if and only if it is connected.

6). Show that each pathwise connected subspace of $\mathbb{R}$ is an (open, half-open or closed) interval.

7). Prove that the subspace of $\mathbb{R}^3$

$$X = \{(x,y,z) \in \mathbb{R}^3 \mid x^2 + y^2 - z^2 = 0 \text{ and } z \neq 0\}$$

is homotopically equivalent to the disjoint union of two circles.

G O O D   L U C K !