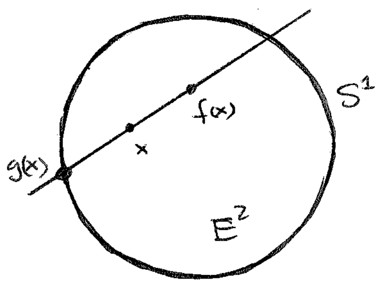
But, since $\pi_1(E^2) = 0$, i_* maps $\pi_1(S^1)$ constantly into zero, and so does $f_* \circ i_*$, and therefore $f_* \circ i_* \neq \mathrm{Id}_{\pi_1(S^1)}$. Contradiction!

<u>Corollary 2:</u> For each continuous map $f : E^2 \to E^2$ there exists a point $x \in E^2$ such that $f(x) = x$.

<u>Sketch of proof:</u> Suppose $\exists\, f : E^2 \to E^2$ continuous with $f(x) \neq x$ for all $x \in E^2$. Then x and $f(x)$ span a unique straight line which intersects the circle S^1 at exactly two points, one of them lying nearer to $f(x)$ and the other one, denoted by $g(x)$, lying nearer to x. It is not hard to show that the resulting map $g : E^2 \to S^1$ is continuous, and that $g(x) = x$ if $x \in S^1$. But the existence of such a map g is impossible by Cor. 1 on p. 18. Contradiction!



<u>Corollary 3:</u> For the torus T^2 we have

$$\pi_1(T^2) \cong \mathbb{Z} \times \mathbb{Z} .$$

(This follows from $T^2 \cong S^1 \times S^1$ and problem 4 in <u>N.</u>)

<u>P.</u> <u>Some final remarks without proof</u>

For higher dimensional spheres one has

<u>Theorem:</u> $\pi_1(S^n) = 0$ for $n > 1$.

Hence in particular $\pi_1(S^2) = 0$, but $\pi_1(T^2) \cong \mathbb{Z} \times \mathbb{Z}$ (Cor. 3. above), therefore <u>the sphere S^2 and the torus T^2 are not homeomorphic</u> (according to problem 3 on p. 17 they are not even homotopy equivalent).

The theorem above shows that the fundamental group cannot distinguish