these groups are called <u>isomorphic.</u>

Groups are the basic objects in algebra. We have the following correspondences:

topology		algebra
topological spaces	———	groups
continuous maps	———	group homomorphisms
homeomorphisms	———	group isomorphisms

N. The fundamental group (II)

In L. we associated to each topological space X (with a basepoint x_o) a group $\pi_1(X,x_o)$. This group, roughly speaking, counts certain holes in the space X. The simpler X is (from the homotopy point of view), the simpler $\pi_1(X,x_o)$ will be in general.

Problem 1: Show that $\pi_1(\mathbb{R}^n,0) = 0$, and that for each $x_o \in E^n$ also $\pi_1(E^n,x_o) = 0$.

For pathwise connected spaces the fundamental group does not essentially depend on the choice of the basepoint:

Problem 2: Let X be a pathwise connected space, and x_o and $\tilde{x}_o$ two points in X. Prove that the groups $\pi_1(X,x_o)$ and $\pi_1(X,\tilde{x}_o)$ are isomorphic. Therefore $\pi_1(X,x_o)$ is, up to isomorphism, independent of the choice of the basepoint x_o, and can be denoted by $\pi_1(X)$.

Now let X and Y be topological spaces with basepoints x_o and y_o resp., and let $f : X \to Y$ be continuous with $f(x_o) = y_o$. Then for each loop $c \in \mathcal{L}(X,x_o)$ the composite loop

$$f \circ c : I \xrightarrow{c} X \xrightarrow{f} Y$$

is an element of $\mathcal{L}(Y,y_o)$. Furthermore if $c,\tilde{c} \in \mathcal{L}(X,x_o)$ are