(resp. rational) numbers, with the usual addition $+$ as composition, form a group denoted by $\mathbb{R}$ (resp. $\mathbb{Q}$).

3). The punctured real line $\mathbb{R} - \{0\}$, together with the usual multiplication $\circ$ as composition, forms a group.

4). The circle S^1, together with the usual complex multiplcation $\circ$, forms a group.

5). If X is a topological space, the set $Aut(X)$ of all homeomorphisms from X onto itself, together with the usual composition as maps, forms a group. ∎

Let G_1 and G_2 be groups. Compose two elements (g_1,g_2) and $(\tilde{g}_1,\tilde{g}_2)$ of the product set $G_1 \times G_2$ as follows:

$$(g_1,g_2) \circ (\tilde{g}_1,\tilde{g}_2) = (g_1 \circ \tilde{g}_1,\ g_2 \circ \tilde{g}_2) .$$

The set $G_1 \times G_2$, together with this composition, forms a group, the <u>product group</u> of G_1 and G_2.

<u>Definition:</u> Let G_1 and G_2 be groups. A map $f : G_1 \to G_2$ is called a <u>group homomorphism</u> if

$$\forall_{g_1,\tilde{g}_1 \in G_1} \quad \underset{\substack{\uparrow \\ \left(\text{composition in } G_1\right)}}{f(g_1 \circ \tilde{g}_1)} = \underset{\substack{\uparrow \\ \left(\text{composition in } G_2\right)}}{f(g_1) \circ f(\tilde{g}_1)} .$$

<u>Examples:</u> The inclusion $i : \mathbb{Z} \to \mathbb{R}$ is a group homomorphism, as is the map

$$\begin{aligned} f : \mathbb{Z} &\longrightarrow \mathbb{Z}_2 \\ \text{even \#'s} &\to 0 \\ \text{odd \#'s} &\to 1 . \end{aligned}$$

<u>Definition:</u> A map $f : G_1 \to G_2$ is <u>group isomorphism</u> if it is a group homomorphism and has an inverse map (which is also a group homomorphism). If there is a group isomorphism between two groups,