group of X relative the basepoint x_o. Define $c_{x_o} \in \mathcal{L}(X,x_o)$ by $c_{x_o}(t) = x_o$ for all $t \in I$, and denote its homotopy class $[c_{x_o}]$ by 0 or $[x_o]$. Also, for $[c] \in \pi_1(X,x_o)$ define $-[c]$ by $-[c] = [c^{-1}]$, where $c^{-1} \in \mathcal{L}(X,x_o)$ is defined by $c^{-1}(t) = c(1-t)$.

Exercise: Prove that the addition of $\pi_1(X,x_o)$ has the following properties

(i) (associativity): $([c]+[\tilde{c}]) + [\tilde{\tilde{c}}] = [c] + ([\tilde{c}]+[\tilde{\tilde{c}}])$ for any three elements $[c], [\tilde{c}], [\tilde{\tilde{c}}] \in \pi_1(X,x_o)$

(ii) for all $[c] \in \pi_1(X,x_o)$ $[c] + 0 = 0 + [c] = [c]$

(iii) for all $[c] \in \pi_1(X,x_o)$ $[c] + (-[c]) = (-[c]) + [c] = 0$

i.e. $\pi_1(X,x_o)$ is a group in the sense below.

M. Groups

Definition: A group is a set G together with a composition $\circ$ which to any two elements $g_1,g_2 \in G$ associates an element $g_1 \circ g_2$ of G, such that

(i) $\forall\, g_1,g_2,g_3 \in G \quad (g_1 \circ g_2) \circ g_3 = g_1 \circ (g_2 \circ g_3)$ (associativity)

(ii) $\exists\, e \in G$ such that $\forall\, g \in G \quad g \circ e = e \circ g = g$
(e is called the zero element or the unit of G)

(iii) $\forall\, g \in G \ \exists\, g^{-1} \in G$ such that $g \circ g^{-1} = g^{-1} \circ g = e$
(g^{-1} is called the inverse of g).

Examples.

0). The group consisting only of its unit and having no other elements is denoted by 0.

1). $\mathbb{Z}_2$ is the group consisting of two elements 0 and 1 with the composition $+$ given by $0 + 0 = 1 + 1 = 0$ and $0 + 1 = 1 + 0 = 1$.

2). The integer numbers $\{0,\pm1,\pm2,\dots\}$, together with the usual addition $+$, form a group denoted by $\mathbb{Z}$. Similarly the real