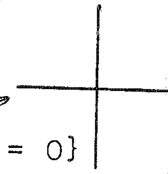
P^n is obtained by "identifying antipodical points" on the sphere S^n. P^n also can be interpreted as the space of lines in $\mathbb{R}^{n+1}$ which go through the origin. Show that P^n is an n-dimensional manifold. How does P^1 look like? How P^2?



5). Show that the "cross" $\{(x,y) \in \mathbb{R}^2 \mid x,y \in (-1,1); x \text{ or } y = 0\}$ is not a manifold.

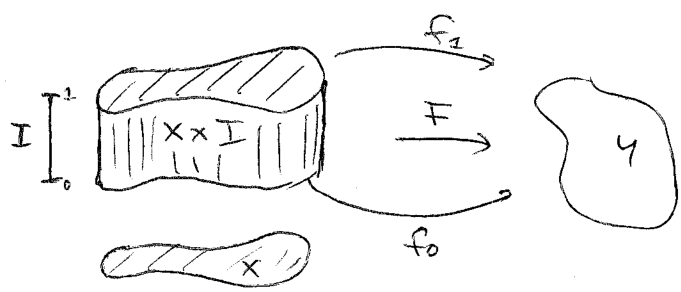
6). Let M be a manifold; $x_o, x_1 \in M$, $x_o \neq x_1$; construct a continuous function $f : M \to \mathbb{R}$ with $f(x_o) = 0$ and $f(x_1) = 1$.

Manifolds are the central objects of study in topology.

J. Homotopy between maps

Definition: Let x,y be topological spaces, and $f_o, f_1 : X \to Y$ continuous maps.

f_o and f_1 are called homotopic (and written "$f_o \sim f_1$") if there is a continuous map



$F : X \times I \to Y$ s.t. $\forall\, x \in X$ $F(x,0) = f_o(x)$ and $F(x,1) = f_1(x)$. (Such a map F is called a homotopy between f_o and f_1).

$\not\vdash$: The notion of homotopy between maps gives an equivalence relation, i.e. for continuous maps $f, f_o, f_1, f_2 : X \to Y$ we have

(i) $f \sim f$ (reflexivity)

(ii) $f_o \sim f_1 \to f_1 \sim f_o$ (symmetry)

(iii) $f_o \sim f_1$ and $f_1 \sim f_2 \to f_o \sim f_2$ (transitivity)

Lemma: Let X, X', Y, Y' be topological spaces and $f_o, f_1 : X \to Y$, $g : X' \to X$, $h : Y \to Y'$ continuous maps. Then if $f_o \sim f_1$, we