<u>Corollary:</u> If $f,g : X \to \mathbb{R}$ are continuous real-valued functions defined on the topological space X, then so are

$$(f+g) : X \to \mathbb{R} \quad\quad \text{and} \quad\quad (f\cdot g) : X \to \mathbb{R}$$
$$x \to f(x)+g(x) \quad\quad\quad\quad x \to f(x)\cdot g(x) \quad .$$

If in addition $\underset{x\in X}{\forall}\ g(x) \neq 0$, then also

$$\frac{f}{g} : X \to \mathbb{R}, \quad x \to \frac{f(x)}{g(x)} \quad \text{is continuous .}$$

<u>Problems:</u>

1.) Check all the statements made on $\mathbb{R}^n$. In particular prove that the three topologies on $\mathbb{R}^n$ introduced above are in fact topologies (i.e. satisfy the axioms (i), (ii) and (iii) of a topology), and that they coincide.

2.) Show that $\mathbb{R}^n$ is Hausdorff, regular, normal and connected. Is $\mathbb{R}^n$ compact?

3.) Show that a subspace of $\mathbb{R}^n$ is compact iff it is closed in $\mathbb{R}^n$ and bounded. (Def.: A subset K of $\mathbb{R}^n$ is bounded if $\exists\ N \in \mathbb{R}$ s.t. $\forall\ x \in K\ \|x\| < N$)

G. <u>Important subspaces of Euclidean space</u>

$I = [0,1] \subset \mathbb{R}$ "unit intervall"

$E^n = \{x \in \mathbb{R}^n \mid \|x\| \leq 1\} \subset \mathbb{R}^n$ "closed unit ball"

$D^n = \{x \in \mathbb{R}^n \mid \|x\| \leq 1\} \subset \mathbb{R}^n$ "open unit ball"

$S^{n-1} = \{x \in \mathbb{R}^n \mid \|x\| = 1\}$ "unit sphere" (of dim. n-1)

$W^n = \{x \in \mathbb{R}^n \mid |||x||| \leq 1\}$ "closed unit cube" in $\mathbb{R}^n$

$I^n = \underbrace{I \times \dots \times I}_{n \text{ times}} = \{x \in \mathbb{R}^n \mid \underset{i\leq n}{\forall}\ 0 \leq x_i \leq 1\}$