is open <--> $\forall\, x_o \in U\ \exists\, \varepsilon > 0$ such that $\forall\, x \in \mathbb{R}^n$ with $d(x,x_o) < \varepsilon$ we have $x \in U$ (i.e. with each $x_o \in U$ U also contains a whole ball around x_o). Similarly we can define the <u>cubic norm $|||\ |||$</u> on $\mathbb{R}^n$ by

$$|||\, x\, ||| \;=\; \operatorname*{Max}_{1\le i\le n} \{|x_i|\} \quad \text{for} \quad x = (x_1,\dots x_n) \in \mathbb{R}^n .$$

This norm also satisfies the norm axioms above, and gives rise to the <u>cubic metric $\tilde{d}$</u> defined by

$$\tilde{d}(x,y) \;=\; |||x - y||| \qquad \text{for} \quad x,y \in \mathbb{R}^n .$$

$\tilde{d}$ has the properties (i), (ii) and (iii) of a metric, and hence defines also a topology on $\mathbb{R}^n$.

<u>Proposition:</u> The three topologies on $\mathbb{R}^n$ introduced so far (i.e. 1.) the metric topology given by d, 2.) the metric topology given by $\tilde{d}$, 3.) the product topology) coincide. Thus in particular a map $f : \begin{array}{l} X \to \mathbb{R}^n \\ x \to f(x)=y=(y_1,..,y_n) \end{array}$ from a topological space X into $\mathbb{R}^n$ is continuous iff the component functions

$$\pi_i \cdot f : \begin{array}{l} X \to \mathbb{R} \\ x \to y_i \end{array} \quad \text{are continuous for all} \quad i \le n .$$

<u>Lemma.</u> The following maps are continuous:

1.) add: $\begin{array}{l} \mathbb{R}^n \times \mathbb{R}^n \to \mathbb{R}^n \\ (x\ ,\ y) \to x + y \end{array}$

2.) mult: $\begin{array}{l} \mathbb{R} \times \mathbb{R}^n \to \mathbb{R}^n \\ (\lambda\ ,\ x) \to \lambda \cdot x \end{array}$

3.) inv: $\begin{array}{l} \mathbb{R} - \{0\} \to \mathbb{R} \\ \lambda \to \frac{1}{\lambda} \end{array}$.