in X. This defines a topology on Y, called the <u>quotient topology</u> of Y (with respect to the map f).

Problem: Prove the following lemma.

L : $f : X \to Y$ is continuous; and a map $g : Y \to Z$ is continuous $\Longleftrightarrow$ $g \circ f : X \to Z$ is continuous.

Example: Let X be a topological space, and $A \subset X$ a subset. Let Y be the set obtained by collapsing A into a point, i.e. Y consists of $\{A\}$ (which forms a single point in Y) and of the points of $X - A$. Let $f : X \to Y$ be the map which leaves the points in $X - A$ unchanged and which maps each point $x \in A$ into the point $\{A\} \in Y$. Then Y with the quotient topology is denoted by $X/_A$.

F. Euclidean space

Throughout this section n is a natural number. We denote by

$$\mathbb{R}^n = \{(x_1, \ldots, x_n) \mid \forall_{i \le n} \quad x_i \in \mathbb{R}\}$$

the set of all ordered sequences of n real numbers. In $\mathbb{R}^n$ we have an addition, a multiplication with real numbers and a scalar product: if $x = (x_1,\ldots,x_n)$ and $y = (y_1,\ldots,y_n)$ are points in $\mathbb{R}^n$ and $\lambda \in \mathbb{R}$, we define:

$$x + y = (x_1+y_1, x_2+y_2, \ldots, x_n+y_n) \in \mathbb{R}^n$$

$$\lambda \cdot x = (\lambda x_1, \lambda x_2, \ldots, \lambda x_n) \in \mathbb{R}^n$$

$$(x, y) = (x_1 y_1 + x_2 y_2 + \ldots . x_n y_n) \in \mathbb{R}^n$$

$$\| x \| = {}_{+}\sqrt{(x,x)} = {}_{+}\sqrt{x_2^2 + x_2^2 + \ldots + x_n^2}$$

We have the following <u>"Schwarz inequality"</u>:

$$|(x,y)| \le \| x \| \cdot \| y \|$$

Therefore, if $x,y \in \mathbb{R}^n - \{0\}$, $\dfrac{(x,y)}{\|x\| \cdot \|y\|}$ lies between -1 and