$(U_j)_{j\in J}$ a collection of open subsets covering X. Prove: $f : X \to Y$ is continuous $\Leftrightarrow$ $\forall_{j\in J}\ f|U_j$ is continuous.

3) Let $f : X \to Y$ be as above, and let A_1 and A_2 be closed subsets of X such that $X = A_1 \cup A_2$. Then: f is continuous $\Leftrightarrow$ $f|A_1$ and $f|A_2$ are continuous.

D. Product spaces

Definition: Let $X_1,\dots X_n$ be topological spaces. A "basic open" subset of the product set

$$X_1 \times X_2 \times \dots \times X_n = \{(x_1,x_2,\dots,x_n) \mid x_1 \in X_1,\dots, x_n \in X_n\}$$

is a set of the form

$$U_1 \times U_2 \times \dots \times U_n = \{(x_1,\dots x_n) \mid \forall i\ x_i \in U_i\} \subset X_1 \times \dots \times X_n$$

where $\forall i \le n$ U_i is open in X_i. A subset of $X_1 \times \dots \times X_n$ is called open if it is any union of such basic open subsets. This defines a topology on $X_1 \times \dots \times X_n$, and $X_1 \times \dots \times X_n$ with this topology is called the product space of $X_1, X_2, \dots$ and X_n.

$\vdash$: For each $i = 1,\dots$ or n the projection map

$$\pi_i : X_1 \times \dots \times X_i \times \dots \times X_n \longrightarrow X_i$$
$$(x_1,x_2,\dots x_i,\dots x_n) \longrightarrow x_i$$

is continuous. Furthermore a map $f : Y \to X_1 \times \dots X_n$ from a topological space Y in the product space is continuous iff $\forall_{1\le i\le n}\ \pi_i \circ f : Y \to X_i$ is continuous.

Problem:

1) Prove that the product of two compact spaces is compact.

E. Quotient spaces

Let X be a topological space and let $f : X \to Y$ be a map onto a set Y. Then we call a subset U of Y open iff $f^{-1}(U)$ is open