

- at $x_0 \in X$ in terms of ϵ and δ ?
- 2) Define a function $f : (0,1) \rightarrow \mathbb{R}$ in the following way: for irrational $x \in (0,1)$ put $f(x) = 0$; and for rational $x \in (0,1)$, write x in the form $x = \frac{p}{q}$ (with p, q relatively prime natural numbers), and put $f(x) = \frac{1}{q}$. Determine all the points $x_0 \in (0,1)$ at which f is continuous.

C. Subspaces

Let A be a subset of a topological space Y .

Def.: A set U in A is called open in A $\leftrightarrow \exists V$ open in Y such that $U = V \cap A$.

The open subsets in A form a topology of A , the "subspace topology" of "induced topology". Henceforth all subsets of topological spaces are considered as topological spaces with the induced topology.

ι : The inclusion map $i : A \hookrightarrow Y$ (with $i(y) = y$) is continuous; furthermore a map $f : X \rightarrow A$ from a topological space X into A is continuous iff $i \circ f : X \rightarrow Y$ is continuous.

Problem:

1) Which of the following is true:

- (i) each subspace of a Hausdorff space is Hausdorff?
- (ii) " " " " compact " " compact?
- (iii) " " " " metric " " metric?
- (iv) " " " " connected " " connected?

Def.: If $f : X \rightarrow Y$ is a map, and $U \subset X$, then the restriction of f to U is denoted by $f|U : U \rightarrow Y$ (and defined by $f|U(x) = f(x)$ for all $x \in U$).

Problems:

2) Let X, Y be topological spaces, $f : X \rightarrow Y$ a map,