Introductory Topology

A. Definition: A topological space is a set X together with a collection $\mathcal{O}$ of "open" subsets of X, such that

(i) $S \in \mathcal{O}$, $\emptyset \in \mathcal{O}$

(ii) any union of open subsets of X is also open

(iii) any finite intersection of open subsets is open

(the collection $\mathcal{O}$ is called the topology of X).

If X is a topological space and $x_o \in X$, then any open subset of X which contains x_o is called a neighborhood of x_o in X.

B. Continuity

Let X,Y be topological spaces and $f : X \to Y$ a map.

Def.: (i) f is continuous at $x_o \in X \iff \forall$ neighborhood V of $f(x_o)$ in Y $\exists$ a neighborhood U of x_o in X such that $f(U) \subset V$.

(ii) $f : X \to Y$ is continuous $\iff \forall\, x_o \in X$ f is continuous at x_o.

$\vdash$: $f : X \to Y$ is continuous $\iff$ the inverse image of each open subset of Y is open in X $\iff$ the inverse image of each closed subset of Y is closed in X.

$\vdash$: If $f : X \to Y$ and $g : Y \to Z$ are continuous maps (X,Y,Z topological spaces), then $g \circ f : X \to Z$ is continuous.

Problems:

1) In the special case that X and Y are metric spaces, how can one express the continuity of a map $f : X \to Y$